In this worksheet, we come back to integrating: \( P \, dx + Q \, dy \) over a path \( C \). A physical interpretation of this integral with appropriate identification is work, or circulation, or flux.

We suggest three methods of evaluating this integral:

- Evaluate the integral directly.

- In case \( F(x,y) = [ P(x,y)dx+ Q(x,y)dy ] \) is "conservative", find the potential \( G \), so that \( \text{grad}(G) = F \). Then, use the Fundamental Theorem of integral calculus.

- In case \( F \) is not conservative and \( C \) is closed, Green's Theorem gives an alternate method for evaluating the integral. What happens if you use Green’s Theorem for “conservative” \( F \) on a closed path \( C \)?

Here are two statements of **Green's Theorem**.

**FLUX:**
The outward flux of a field \( E = Pi + Qj \) across a closed curve \( C \) equals the double integral of the divergence of \( F \) over the region enclosed by \( C \).

**CIRCULATION:**
The counterclockwise circulation of a field \( E = Pi +Qj \) around a closed curve \( C \) in a plane, equals the double integral of the kth component of the curl of \( E \) over the region bounded by \( C \).

**EXAMPLES:**
(1) \( F(x, y) = [\ln(x^2+y^2)dx + \ln(x^2+y^2)dy] \), \( C \) is the boundary of the half washer with radii: 1 and 2.
(2) \( F(x, y) = [xy \, dx + y^2+x^2 \, dy] \), where \( C \) is the triangle \([0,0], [3,0], [2,2] \).
(3) \( F(x, y) = [y^3+2y \, dx + 3y^2x \, dy] \), \( C \) is the circle with center \([0,0] \) and radius 3.